DENGUE TRANSMISSION MODEL WITH THE DIFFERENT INCUBATION RATE FOR EACH SEASON

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Abstract

A model is used for describing the transmission of dengue disease. This disease is occurred by biting of the infected *Aedes* mosquitoes. Dengue outbreak is found in the rainy, winter and summer seasons. Each season has the different dengue outbreaks and they are depend on the temperature of the environment. The standard dynamical modeling method is used in this study. The SEIR (S = susceptible , E= exposed , I = infected and R = recovered) model is used. We use conditions of parameters for determining the local stability of disease free equilibrium state and disease endemic equilibrium state. The basic reproductive number of disease is found. The disease free state is local stability when $R_0 < 1$ .The endemic disease state is local stability when $R_0 > 1$. The control of this disease is discussed in this paper.

Keywords: Dengue disease, local stability, transmission model ,SEIR model, season, incubation rate.

Introduction

Transmission of dengue virus may serve to retain viral pathogen in nature during inter-epidemic periods of the disease [2]. Dengue fever (DF) , Dengue Haemorrhagic Fever (DHF) and Dengue Shock Syndrome (DSS) are three types of dengue disease. There are four serotypes of dengue virus, namely DEN-1, DEN-2, DEN-3 and DEN-4. Dengue disease can not be directly transmitted between the people. Transmission is occurred by biting of the female *Aedes* mosquito. The development of virus and mosquito are affected by the climatic factors. When infected mosquito bites the human , thus the human are exposed and infected. Symptoms of dengue fever are depend on age. In older children, teenagers and adults, the most common symptoms of dengue disease are fever that comes on quickly and lasts two to seven days but this usually is not severe, muscle and joint pain, a red rash that starts on chest, back or stomach and spreads to your limbs and face, feeling sick, vomiting and diarrhea. The symptoms of dengue fever usually begin between five and eight days after each person be get bitten by an infected mosquito. Dengue fever is caused by a type of virus called a *flavivirus*, which is transmitted by infected female *Aedes* mosquitoes. We can catch the virus if we get be bitten by an infected mosquito. Mosquitoes become infected when they bite an infected person and are able to pass on the virus for the rest of their life. In Thailand, the annual estimations of dengue fever are depend on the season. The *Aedes aegypti* is the principal transmitter of Dengue fever in Thailand but it also transmits Chikungunya fever, yellow fever and Filariasis among other diseases. The Aedes aegypti prefers feed during

daylight hours. They adapt very easily to human surroundings and will lay their eggs where there is water, including plastic containers, bins, plant pots etc. Thailand's rainy season, starting from May through September, is also the high risk period for dengue fever, a potentially serious condition is the most prevalent in tropical countries. *Aedes aegypti* mosquitoes carry the virus that causes dengue fever, and they infect 50 million people a year, including 500,000 serious cases requiring hospitalization[8].The moisture content ,temperature,season and rainfall are influence to the mosquito development. Dengue infection is endemic in Thailand. From the data of Dengue cases in 1999 – 2010, we can see that most dengue patients are occurred in rainy season. We can see as shown in figure 1.
The purpose of this paper is to incorporate this feature into the SEIR model. Models keep track of an individual's infection-age for particular diseases, for instance tubercolosis[3]. Esteva and Vargas developed a model for dengue disease transmission and included the dynamics of the *Aedes aegypti mosquito into standard SIR* (susceptible – infective-recovered) epidemic model [5]. In this paper , we used SEIR model for analyzing and finding the method to decrease the outbreak of this disease. We analyze dengue model of seasonality compartment (rainy season, winter season and summer season).

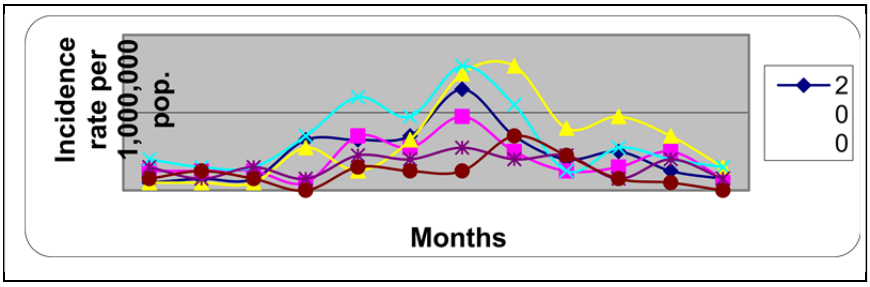

Figure 1 Reported cases of Dengue disease per 100,000 population in Thailand during year 2005 and 2010 .

Methodology

The mathematical modeling for dengue disease describes the relevance of human and mosquito population. In this study, we assume that the total human and mosquito population have constant sizes. The human population is divided into susceptible , exposed , infected and recovered classes for the first model. The mosquito population is divided into susceptible , exposed and infected classes because the mosquito never recover from infection. The model considers transmission of dengue virus in human and mosquito population by model :
The dynamics of human population are given by

$$\frac{d}{dt}S_{hr} = C_r N_{Tr} - \lambda_d S_{hr} - \lambda_h S_{hr} - \delta \frac{K_{v\to hr}}{N_{Tr}+g} I_{rr} S_{hr} \tag{1}$$

$$\frac{d}{dt}E_{hr} = \delta \frac{K_{v\to hr}}{N_{Tr}+g} I_{rr} S_{hr} - \lambda_d E_{hr} - \lambda_h E_{hr} - \alpha_{hr} E_{hr} \tag{2}$$

$$\frac{d}{dt}I_{hr} = \alpha_{hr} - \lambda_d I_{hr} - \lambda_h I_{hr} - \rho I_{hr} \tag{3}$$

$$\frac{d}{dt}R_{hr} = \rho I_{hr} - \lambda_d R_{hr} - \lambda_h R_{hr} \tag{4}$$

$$\frac{d}{dt}S_{hw} = C_w N_{Tw} - \lambda_d S_{hw} - \lambda_h S_{hw} - \delta \frac{K_{v\to hw}}{N_{Tw}+g} I_{rw} S_{hw} \tag{5}$$

$$\frac{d}{dt}E_{hw} = \delta \frac{K_{v \to hw}}{N_{Tw} + g} I_{rw} S_{hw} - \lambda_d E_{hw} - \lambda_h E_{hw} - \alpha_{hw} E_{hw} \quad (6)$$

$$\frac{d}{dt}I_{hw} = \alpha_{hw} - \lambda_d I_{hw} - \lambda_h I_{hw} - \rho I_{hw} \quad (7)$$

$$\frac{d}{dt}R_{hw} = \rho I_{hw} - \lambda_d R_{hw} - \lambda_h R_{hw} \quad (8)$$

$$\frac{d}{dt}S_{hs} = C_s N_{Ts} - \lambda_d S_{hs} - \lambda_h S_{hs} - \delta \frac{K_{v \to hs}}{N_{Ts} + g} I_{rs} S_{hs} \quad (9)$$

$$\frac{d}{dt}E_{hs} = \delta \frac{K_{v \to hs}}{N_{Ts} + g} I_{rs} S_{hs} - \lambda_d E_{hs} - \lambda_h E_{hs} - \alpha_{hs} E_{hs} \quad (10)$$

$$\frac{d}{dt}I_{hs} = \alpha_{hs} - \lambda_d I_{hs} - \lambda_h I_{hs} - \rho I_{hs} \quad (11)$$

$$\frac{d}{dt}R_{hs} = \rho I_{hs} - \lambda_d R_{hs} - \lambda_h R_{hs} \quad (12)$$

We define

S_{hr} is the number of susceptible human population in rainy season,
E_{hr} is the number of exposed human population in rainy season,
I_{hr} is the number of infectious human population in rainy season,
R_{hr} is the number of recovered human population in rainy season,
S_{hw} is the number of susceptible human population in winter season,
E_{hw} is the number of exposed human population in winter season,
I_{hw} is the number of infectious human population in winter season,
R_{hw} is the number of recovered human population in winter season,
S_{hs} is the number of susceptible human population in summer season,
E_{hs} is the number of exposed human population in summer season,
I_{hs} is the number of infectious human population in summer season,
R_{hs} is the number of recovered human population in summer season

The dynamics of the mosquito population are given by :

$$\frac{d}{dt}S_{vr} = Z_r - \delta \frac{K_{hr \to v}}{N_{Tr} + g} I_{hr} S_{vr} - \lambda_v S_{vr} \quad (13)$$

$$\frac{d}{dt}E_{vr} = \delta \frac{K_{hr \to v}}{N_{Tr} + g} I_{hr} S_{vr} - \lambda_v E_{vr} - \beta_{vr} E_{vr} \quad (14)$$

$$\frac{d}{dt}I_{vr} = \beta_{vr} E_{vr} - \lambda_v I_{vr} \quad (15)$$

$$\frac{d}{dt}S_{vw} = Z_w - \delta \frac{K_{hw \to v}}{N_{Tw} + g} I_{hw} S_{vw} - \lambda_v S_{vw} \quad (16)$$

$$\frac{d}{dt}E_{vw} = \delta \frac{K_{hw \to v}}{N_{Tw} + g} I_{hw} S_{vw} - \lambda_v E_{vw} - \beta_{vw} E_{vw} \quad (17)$$

$$\frac{d}{dt}I_{vw} = \beta_{vw} E_{vw} - \lambda_v I_{vw} \quad (18)$$

$$\frac{d}{dt}S_{vs} = Z_s - \delta \frac{K_{hs \to v}}{N_{Ts} + g} I_{hs} S_{vs} - \lambda_v S_{vs} \quad (19)$$

$$\frac{d}{dt}E_{vs} = \delta \frac{K_{hs\to v}}{N_{Ts}+g} I_{hs}S_{vs} - \lambda_v E_{vs} - \beta_{vs}E_{vs} \tag{20}$$

$$\frac{d}{dt}I_{vs} = \beta_{vs}E_{vs} - \lambda_v I_{vs} \tag{21}$$

We define

S_{vr} is the number of susceptible mosquito population in rainy season,

E_{vr} is the number of exposed mosquito population in rainy season,

I_{vr} is the number of infectious mosquito population in rainy season,

S_{vw} is the number of susceptible mosquito population in winter season,

E_{vw} is the number of exposed mosquito population in winter season,

I_{vw} is the number of infectious mosquito population in winter season,

S_{vs} is the number of susceptible mosquito population in summer season,

E_{vs} is the number of exposed mosquito population in summer season,

I_{vs} is the number of infectious mosquito population in summer season.

Where the parameters are defined as follows :

N_{Tr} is the total human population in rainy season,

N_{Tw} is the total human population in winter season,

N_{Ts} is the total human population in summer season,

N_{Vr} is the total mosquito population in rainy season,

N_{Vw} is the total mosquito population in winter season,

N_{Vs} is the total mosquito population in summer season,

λ_h is the natural death rate of human population,

λ_d is the death rate of human population due to the disease,

λ_v is the death rate of mosquito population,

K is the birth rate of human population,

$K_{v\to hr}$ is the transmission probability of dengue disease from mosquito to human in rainy season,

$K_{v\to hw}$ is the transmission probability of dengue disease from mosquito to human in winter season ,

$K_{v\to hs}$ is the transmission probability of dengue disease from mosquito to human in summer season,

$K_{hr\to v}$ is the transmission probability of dengue disease from human to mosquito in rainy season,

$K_{hw\to v}$ is the transmission probability of dengue disease from human to mosquito in winter season,

$K_{hs\to v}$ is the transmission probability of dengue disease from human to mosquito in summer season,

α_{hr} is the incubation rate of human population in rainy season,

α_{hw} is the incubation rate of human population in winter season,

α_{hs} is the incubation rate of human population in summer season,

α_{vr} is the incubation rate of mosquito population in rainy season
α_{vw} is the incubation rate of mosquito population in winter season
α_{vs} is the incubation rate of mosquito population in summer season,
ρ is the recovery rate of human population,
δ is the biting rate of mosquito population,
g is the number of other animals available as blood sources.

We suppose that $N_{Hr} = S_{hr} + E_{hr} + I_{hr} + R_{hr}$, $N_{Hw} = S_{hw} + E_{hw} + I_{hw} + R_{hw}$, $N_{Hs} = S_{hs} + E_{hs} + I_{hs} + R_{hs}$, $N_{Vr} = S_{vr} + E_{vr} + I_{vr}$, $N_{Vw} = S_{vw} + E_{vw} + I_{vw}$ and $N_{Vs} = S_{vs} + E_{vs} + I_{vs}$

we assume the total human and mosquito populations have constant sizes $\frac{dN_{hr}}{dt} = 0$, $\frac{dN_{vr}}{dt} = 0$ in rainy season, $\frac{dN_{hw}}{dt} = 0$, $\frac{dN_{vw}}{dt} = 0$ in winter season and $\frac{dN_{hs}}{dt} = 0$, $\frac{dN_{vs}}{dt} = 0$ in summer season.

$$\overline{S_{hr}} = \frac{S_{hr}}{N_{Tr}},\ \overline{E_{hr}} = \frac{E_{hr}}{N_{Tr}},\ \overline{I_{hr}} = \frac{I_{hr}}{N_{Tr}},\ \overline{R_{hr}} = \frac{R_{hr}}{N_{Tr}},$$

$$\overline{S_{hw}} = \frac{S_{hw}}{N_{Tw}},\ \overline{E_{hw}} = \frac{E_{hw}}{N_{Tw}},\ \overline{I_{hw}} = \frac{I_{hw}}{N_{Tw}},\ \overline{R_{hw}} = \frac{R_{hw}}{N_{Tw}},$$

$$\overline{S_{hs}} = \frac{S_{hs}}{N_{Ts}},\ \overline{E_{hs}} = \frac{E_{hs}}{N_{Ts}},\ \overline{I_{hs}} = \frac{I_{hs}}{N_{Ts}},\ \overline{R_{hs}} = \frac{R_{hs}}{N_{Ts}}$$

$$\overline{S_{vr}} = \frac{S_{vr}}{N_{Vr}},\ \overline{S_{vw}} = \frac{S_{vw}}{N_{Vw}},\ \overline{S_{vs}} = \frac{S_{vs}}{N_{Vs}},\ \overline{E_{vr}} = \frac{E_{vr}}{N_{Vr}},\ \overline{E_{vw}} = \frac{E_{vw}}{N_{Vw}},\ \overline{E_{vs}} = \frac{E_{vs}}{N_{Vs}}$$

$$\overline{I_{vr}} = \frac{I_{vr}}{N_{Vr}},\ \overline{I_{vw}} = \frac{I_{vw}}{N_{Vw}},\ \overline{I_{vs}} = \frac{I_{vs}}{N_{Vs}}$$

These give

$$\frac{d}{dt}\overline{S_{hr}} = (\lambda_d + \lambda_h) - (\lambda_d + \lambda_h + \delta\frac{K_{v\to hr}}{N_{Tr}+g}\overline{I_{vr}}\,N_{Vr})\overline{S_{hr}} \tag{22}$$

$$\frac{d}{dt}\overline{E_{hr}} = \delta\frac{K_{v\to hr}}{N_{Tr}+g}\overline{I_{vr}}\,N_{Vr}\overline{S_{hr}} - (\alpha_{hr} + \lambda_d + \lambda_h)\overline{E_{hr}} \tag{23}$$

$$\frac{d}{dt}\overline{I_{hr}} = \alpha_{hr}\overline{E_{hr}} - (\lambda_d + \lambda_h + \rho)\overline{I_{hr}} \tag{24}$$

$$\frac{d}{dt}\overline{S_{hw}} = (\lambda_d + \lambda_h) - (\lambda_d + \lambda_h + \delta\frac{K_{v\to hw}}{N_{Tw}+g}\overline{I_{vw}}\,N_{Vw})\overline{S_{hw}} \tag{25}$$

$$\frac{d}{dt}\overline{E_{hw}} = \delta\frac{K_{v\to hw}}{N_{Tw}+g}\overline{I_{vw}}\,N_{Vw}\overline{S_{hw}} - (\alpha_{hw} + \lambda_d + \lambda_h)\overline{E_{hw}} \tag{26}$$

$$\frac{d}{dt}\overline{I_{hw}} = \alpha_{hw}\overline{E_{hw}} - (\lambda_d + \lambda_h + \rho)\overline{I_{hw}} \tag{27}$$

$$\frac{d}{dt}\overline{S_{hs}} = (\lambda_d + \lambda_h) - (\lambda_d + \lambda_h + \delta\frac{K_{v\to hs}}{N_{Ts}+g}\overline{I_{vs}}\,N_{Vs})\overline{S_{hs}} \tag{28}$$

$$\frac{d}{dt}\overline{E_{hs}} = \delta\frac{K_{v\to hs}}{N_{Ts}+g}\overline{I_{vs}}\,N_{Vs}\overline{S_{hs}} - (\alpha_{hs} + \lambda_d + \lambda_h)\overline{E_{hs}} \tag{29}$$

$$\frac{d}{dt}\overline{I_{hs}} = \alpha_{hs}\overline{E_{hs}} - (\lambda_d + \lambda_h + \rho)\overline{I_{hs}} \tag{30}$$

$$\frac{d}{dt}\overline{E_{vr}} = \delta \frac{K_{hr\rightarrow v}}{N_{Tr}+g}\overline{I_{hr}}\, N_{Tr}\overline{Sv} - (\lambda_v + \beta_{vr})\overline{E_{vr}} \quad (31)$$

$$\frac{d}{dt}\overline{I_{vr}} = \beta_{vr}\overline{E_{vr}} - \lambda_v \overline{I_{vr}} \quad (32)$$

$$\frac{d}{dt}\overline{E_{vw}} = \delta \frac{K_{v\rightarrow hw}}{N_{Tw}+g}\overline{I_{hw}}\, N_{Tw}\overline{S_{vr}} - (\lambda_v + \beta_{vw})\overline{E_{vw}} \quad (33)$$

$$\frac{d}{dt}\overline{I_{vw}} = \beta_{vw}\overline{E_{vw}} - \lambda_v \overline{I_{vw}} \quad (34)$$

$$\frac{d}{dt}\overline{E_{vs}} = \delta \frac{K_{v\rightarrow hs}}{N_{Ts}+g}\overline{I_{hs}}\, N_{Ts}\overline{S_{vs}} - (\lambda_v + \beta_{vs})\overline{E_{vs}} \quad (35)$$

$$\frac{d}{dt}\overline{I_{vs}} = \beta_{vs}\overline{E_{vs}} - \lambda_v \overline{I_{vs}} \quad (36)$$

R_{hr}, R_{hw}, R_{hs} and S_{vr}, S_{vw}, S_{vs} can be obtained from conditions $S_{hr} + E_{hr} + I_{hr} + R_{hr} = 1$, $S_{hw} + E_{hw} + I_{hw} + R_{hw} = 1$, $S_{hs} + E_{hs} + I_{hs} + R_{hs} = 1$ and $S_{vr} + E_{vr} + I_{vr} = 1$, $S_{vw} + E_{vw} + I_{vw} = 1$, $S_{vs} + E_{vs} + I_{vs} = 1$.

Analysis of the Mathematical Model

The equilibrium points are found by setting the right hand side of (22) – (36) equal to zero. This gives

1)The disease free equilibrium point $M_1 = (1,0,0,0,0,1,0,0,0,0,1,0,0,0,0)$

2)The endemic disease equilibrium point

$M_2 = (S^*_{hr}, E^*_{hr}, I^*_{hr}, E^*_{vr}, I^*_{vr}, S^*_{hw}, E^*_{hw}, I^*_{hw}, E^*_{vw}, I^*_{vw}, S^*_{hs}, E^*_{hs}, I^*_{hs}, E^*_{vs}, I^*_{vs})$,

where

$$S^*_{hr} = \frac{(\lambda_d+\lambda_h)}{(\lambda_d+\lambda_h) + \dfrac{I^*_{hr}N_{Tr}N_{Vr}\beta_{vr}\delta K_{hr\rightarrow v}\delta K_{v\rightarrow hr}}{(N_{Tr}+g)(\beta_{vr}+\lambda_v)(I^*_{hr}N_{Tr}\delta K_{hr\rightarrow v}+(N_{Tr}+g)\lambda_v)}},$$

$$E^*_{hr} = \frac{I^*_{hr}N_{Vr}\beta_{vr}N_{Tr}\delta K_{hr\rightarrow v}\delta K_{v\rightarrow hr}(\lambda_d+\lambda_h)}{(N_{Tr}+g)(\alpha_{hr}+\lambda_d+\lambda_h)(\beta_{vr}+\lambda_v)(I^*_{hr}N_{Tr}\delta K_{hr\rightarrow v}+(N_{Tr}+g)\lambda_v)(\lambda_d+\lambda_h+\dfrac{I^*_{hr}N_{Vr}\beta_{vr}N_{Tr}\delta K_{hr\rightarrow v}\delta K_{v\rightarrow hr}}{(N_{Tr}+g)(\beta_{vr}+\lambda_v)(I^*_{hr}N_{Tr}\delta K_{hr\rightarrow v}+(N_{Tr}+g)\lambda_v)}))}$$

$$, \; I^*_{hr} = \frac{(\lambda_d+\lambda_h)(-(N_{Tr}N_{Vr}\alpha_{hr}\beta_{vr}\delta K_{hr\rightarrow v}\delta K_{v\rightarrow hr}+(N_{Tr}+g)^2(\rho+\lambda_d+\lambda_h)(\alpha_{hr}+\lambda_d+\lambda_h)\lambda_v(\beta_{vr}+\lambda_v)))}{(N_{Tr}\delta K_{hr\rightarrow v}(\rho+\lambda_d+\lambda_h)(\alpha_{hr}+\lambda_d+\lambda_h)(N_{Vr}\beta_{vr}\delta K_{v\rightarrow hr}+(N_{Tr}+g)(\lambda_d+\lambda_h)(\beta_{vr}+\lambda_v)))},$$

$$E^*_{vr} = \frac{I^*_{hr}N_{Tr}\delta K_{hr\rightarrow v}\lambda_v}{(\beta_{vr}+\lambda_v)(I^*_{hr}N_{Tr}\delta K_{hr\rightarrow v}+(N_{Tr}+g)\lambda_v)}, \; I^*_{vr} = \frac{I^*_{hr}N_{Tr}\delta K_{hr\rightarrow v}\beta_{vr}}{(\beta_{vr}+\lambda_v)(I^*_{hr}N_{Tr}\delta K_{hr\rightarrow v}+(N_{Tr}+g)\lambda_v)}$$

$$S^*_{hw} = \frac{(\lambda_d+\lambda_h)}{(\lambda_d+\lambda_h) + \dfrac{I^*_{hw}N_{Tw}N_{Vw}\beta_{vw}\delta K_{hw\rightarrow v}\delta K_{v\rightarrow hw}}{(N_{Tw}+g)(\beta_{vw}+\lambda_v)(I^*_{hw}N_{Tw}\delta K_{hw\rightarrow v}+(N_{Tw}+g)\lambda_v)}},$$

$$E^*_{hw} = \frac{I^*_{hw}N_{Vw}\beta_{vw}N_{Tw}\delta K_{hw\rightarrow v}\delta K_{v\rightarrow hw}(\lambda_d+\lambda_h)}{(N_{Tw}+g)(\alpha_{hw}+\lambda_d+\lambda_h)(\beta_{vw}+\lambda_v)(I^*_{hw}N_{Tw}\delta K_{hw\rightarrow v}+(N_{Tw}+g)\lambda_v)(\lambda_d+\lambda_h+\dfrac{I^*_{hw}N_{Vw}\beta_{vw}N_{Tw}\delta K_{hw\rightarrow v}\delta K_{v\rightarrow hw}}{(N_{Tw}+g)(\beta_{vw}+\lambda_v)(I^*_{hw}N_{Tw}\delta K_{hw\rightarrow v}+(N_{Tw}+g)\lambda_v)}))}$$

$$, \; I^*_{hw} = \frac{(\lambda_d+\lambda_h)(-(N_{Tw}N_{Vw}\alpha_{hw}\beta_{vw}\delta K_{hw\rightarrow v}\delta K_{v\rightarrow hw}+(N_{Tw}+g)^2(\rho+\lambda_d+\lambda_h)(\alpha_{hw}+\lambda_d+\lambda_h)\lambda_v(\beta_{vw}+\lambda_v)))}{(N_{Tw}\delta K_{hw\rightarrow v}(\rho+\lambda_d+\lambda_h)(\alpha_{hw}+\lambda_d+\lambda_h)(N_{Vw}\beta_{vw}\delta K_{v\rightarrow hw}+(N_{Tw}+g)(\lambda_d+\lambda_h)(\beta_{vw}+\lambda_v)))},$$

$$E^*_{vw} = \frac{I^*_{hw}N_{Tw}\delta K_{hw\rightarrow v}\lambda_v}{(\beta_{vw}+\lambda_v)(I^*_{hw}N_{Tw}\delta K_{hw\rightarrow v}+(N_{Tw}+g)\lambda_v)}, \; I^*_{vw} = \frac{I^*_{hw}N_{Tw}\delta K_{hw\rightarrow v}\beta_{vw}}{(\beta_{vw}+\lambda_v)(I^*_{hw}N_{Tw}\delta K_{hw\rightarrow v}+(N_{Tw}+g)\lambda_v)}$$

$$S^*_{hs} = \frac{(\lambda_d + \lambda_h)}{(\lambda_d + \lambda_h) + \frac{I^*_{hs} N_{Ts} N_{Vs} \beta_{vs} \delta K_{hs\to v} \delta K_{v\to hs}}{(N_{Ts}+g)(\beta_{vs}+\lambda_v)(I^*_{hs} N_{Ts} \delta K_{hs\to v} + (N_{Ts}+g)\lambda_v)}},$$

$$E^*_{hs} = \frac{I^*_{hs} N_{Vs} \beta_{vs} N_{Ts} \delta K_{hs\to v} \delta K_{v\to hs} (\lambda_d+\lambda_h)}{(N_{Ts}+g)(\alpha_{hs}+\lambda_d+\lambda_h)(\beta_{vs}+\lambda_v)(I^*_{hs} N_{Ts}\delta K_{hs\to v} + (N_{Ts}+g)\lambda_v)(\lambda_d+\lambda_h + \frac{I^*_{hs} N_{Vs}\beta_{vs} N_{Ts}\delta K_{hs\to v}\delta K_{v\to hs}}{(N_{Ts}+g)(\beta_{vs}+\lambda_v)(I^*_{hs} N_{Ts}\delta K_{hs\to v} + (N_{Ts}+g)\lambda_v)}))}$$

$$, \quad I^*_{hs} = \frac{(\lambda_d+\lambda_h)(-(N_{Ts} N_{Vs} \alpha_{hs} \beta_{vs} \delta K_{hs\to v} \delta K_{v\to hs} + (N_{Ts}+g)^2(\rho+\lambda_d+\lambda_h)(\alpha_{hs}+\lambda_d+\lambda_h)\lambda_v(\beta_{vs}+\lambda_v)))}{(N_{Ts}\delta K_{hs\to v}(\rho+\lambda_d+\lambda_h)(\alpha_{hs}+\lambda_d+\lambda_h)(N_{Vs}\beta_{vs}\delta K_{v\to hs} + (N_{Ts}+g)(\lambda_d+\lambda_h)(\beta_{vs}+\lambda_v)))},$$

$$E^*_{vs} = \frac{I^*_{hs} N_{Ts} \delta K_{hs\to v} \lambda_v}{(\beta_{vs}+\lambda_v)(I^*_{hs} N_{Ts}\delta K_{hs\to v} + (N_{Ts}+g)\lambda_v)}, \quad I^*_{vs} = \frac{I^*_{hs} N_{Ts}\delta K_{hs\to v}\beta_{vs}}{(\beta_{vs}+\lambda_v)(I^*_{hs} N_{Ts}\delta K_{hs\to v} + (N_{Ts}+g)\lambda_v)}$$

and $R_0 > 1, R_0 =$

$$\left[\left(\frac{\delta^2 K_{hr\to v} K_{v\to hr} N_{tr} N_{vr} \alpha_{hr} \beta_{vr}}{(g+N_{tr})^2 (\rho+\lambda_d+\lambda_h)(\alpha_{hr}+\lambda_d+\lambda_h)\lambda_v(\beta_{vr}+\lambda_v)}\right) + \left(\frac{\delta^2 K_{hw\to v} K_{v\to hw} N_{tw} N_{vw} \alpha_{hw} \beta_{vw}}{(g+N_{tw})^2 (\rho+\lambda_d+\lambda_h)(\alpha_{hw}+\lambda_d+\lambda_h)\lambda_v(\beta_{vw}+\lambda_v)}\right) + \left(\frac{\delta^2 K_{hs\to v} K_{v\to hs} N_{ts} N_{vw} \alpha_{hs} \beta_{vs}}{(g+N_{ts})^2 (\rho+\lambda_d+\lambda_h)(\alpha_{hs}+\lambda_d+\lambda_h)\lambda_v(\beta_{vs}+\lambda_v)}\right)\right]$$

B. Stability

The stability of each equilibrium point is determined from linearizing equation (13) – (21) about equilibrium point examining the eigenvalues of the resulting Jacobian matrix. We now consider the eigenvalues of the Jacobian matrix at each equilibrium point. From equation (13)-(21) ,we can write in the matrix form as follows:

$$J_{E_1} = \begin{pmatrix} -(\frac{\delta K_{v\to hr}}{(N_{Tr}+g)} I^*_{vr} N_{Vr} + \lambda_d + \lambda_h) & 0 & 0 & 0 & -(\frac{\delta K_{v\to hr}}{(N_{Tr}+g)} N_{Vr} S^*_{hr}) \\ \frac{\delta K_{v\to hr}}{(N_{Tr}+g)} I^*_{vr} N_{Vr} & -(\lambda_d+\lambda_h+\alpha_{hr}) & 0 & 0 & (\frac{\delta K_{v\to hr}}{(N_{Tr}+g)} N_{Vr} S^*_{hr}) \\ 0 & \alpha_{hr} & -(\lambda_d+\lambda_h+\rho) & 0 & 0 \\ 0 & 0 & (\frac{\delta K_{hr\to v}}{(N_{Tr}+g)} N_{Tr} S^*_{Vr}) & -(\lambda_v+\beta_{Vr}) & 0 \\ 0 & 0 & 0 & \beta_{Vr} & -\lambda_v \end{pmatrix}$$

$$J_{E_2} = \begin{pmatrix} -(\frac{\delta K_{v\to hw}}{(N_{Tw}+g)} I^*_{vw} N_{Vw} + \lambda_d + \lambda_h) & 0 & 0 & 0 & -(\frac{\delta K_{v\to hw}}{(N_{Tw}+g)} N_{Vw} S^*_{hw}) \\ \frac{\delta K_{v\to hw}}{(N_{Tw}+g)} I^*_{vw} N_{Vw} & -(\lambda_d+\lambda_h+\alpha_{hw}) & 0 & 0 & (\frac{\delta K_{v\to hw}}{(N_{Tw}+g)} N_{Vw} S^*_{hw}) \\ 0 & \alpha_{hw} & -(\lambda_d+\lambda_h+\rho) & 0 & 0 \\ 0 & 0 & (\frac{\delta K_{hw\to v}}{(N_{Tw}+g)} N_{Tw} S^*_{Vw}) & -(\lambda_v+\beta_{Vw}) & 0 \\ 0 & 0 & 0 & \beta_{Vw} & -\lambda_v \end{pmatrix}$$

$$J_{E_3} = \begin{pmatrix} -(\frac{\delta K_{v\to hs}}{(N_{Ts}+g)} I^*_{vs} N_{Vs} + \lambda_d + \lambda_h) & 0 & 0 & 0 & -(\frac{\delta K_{v\to hs}}{(N_{Ts}+g)} N_{Vs} S^*_{hs}) \\ \frac{\delta K_{v\to hs}}{(N_{Ts}+g)} I^*_{vs} N_{Vs} & -(\lambda_d+\lambda_h+\alpha_{hs}) & 0 & 0 & (\frac{\delta K_{v\to hs}}{(N_{Ts}+g)} N_{Vs} S^*_{hs}) \\ 0 & \alpha_{hs} & -(\lambda_d+\lambda_h+\rho) & 0 & 0 \\ 0 & 0 & (\frac{\delta K_{hs\to v}}{(N_{Ts}+g)} N_{Ts} S^*_{Vs}) & -(\lambda_v+\beta_{Vs}) & 0 \\ 0 & 0 & 0 & \beta_{Vs} & -\lambda_v \end{pmatrix}$$

The eigenvalues (A) are the solution of the Characteristic equation

$$\det(J - BI_5) = 0$$

where J is the Jacobian matrix evaluated at the equilibrium point . I is the identity matrix.

C. Disease free state

Equilibrium point of disease free state $E_{1,2,3}$ = (1,0,0,0,0,1,0,0,0,0,1,0,0,0,0) has eigenvalues as follows:

$$(-B-\lambda_d-\lambda_h)\left(-\frac{\delta^2 K_{hr\to v}K_{v\to hr}N_{Tr}N_{Vr}S_{Vr}\alpha_{hr}\beta_{vr}}{(N_{Tr}+g)^2}+(-B-\rho-\lambda_d-\lambda_h)(-B-\alpha_{hr}-\lambda_d-\lambda_h)(-B-\beta_{vr}-\lambda_v)\right)$$

The eigenvalues are

$B=-\lambda_d-\lambda_h$ and the remaining 4 eigenvalues are the solutions of

$$\left(-\frac{\delta^2 K_{hr\to v}K_{v\to hr}N_{Tr}N_{Vr}S_{Vr}\alpha_{hr}\beta_{vr}}{(N_{Tr}+g)^2}+(-B-\rho-\lambda_d-\lambda_h)(-B-\alpha_{hr}-\lambda_d-\lambda_h)(-B-\beta_{vr}-\lambda_v)\right)=0$$

$$(-B-\lambda_d-\lambda_h)\left(-\frac{\delta^2 K_{hw\to v}K_{v\to hw}N_{Tw}N_{Vw}S_{Vw}\alpha_{hw}\beta_{vw}}{(N_{Tw}+g)^2}+(-B-\rho-\lambda_d-\lambda_h)(-B-\alpha_{hw}-\lambda_d-\lambda_h)(-B-\beta_{vw}-\lambda_v)\right)$$

The eigenvalues are

$B=-\lambda_d-\lambda_h$ and the remaining 4 eigenvalues are the solutions of

$$\left(-\frac{\delta^2 K_{hw\to v}K_{v\to hw}N_{Tw}N_{Vw}S_{Vw}\alpha_{hw}\beta_{vw}}{(N_{Tw}+g)^2}+(-B-\rho-\lambda_d-\lambda_h)(-B-\alpha_{hw}-\lambda_d-\lambda_h)(-B-\beta_{vw}-\lambda_v)\right)=0$$

$$(-B-\lambda_d-\lambda_h)\left(-\frac{\delta^2 K_{hs\to v}K_{v\to hs}N_{Ts}N_{Vs}S_{Vs}\alpha_{hs}\beta_{vs}}{(N_{Ts}+g)^2}+(-B-\rho-\lambda_d-\lambda_h)(-B-\alpha_{hs}-\lambda_d-\lambda_h)(-B-\beta_{vs}-\lambda_v)\right)$$

The eigenvalues are

$B=-\lambda_d-\lambda_h$ and the remaining 4 eigenvalues are the solutions of

$$\left(-\frac{\delta^2 K_{hs\to v}K_{v\to hs}N_{Ts}N_{Vs}S_{Vs}\alpha_{hs}\beta_{vs}}{(N_{Ts}+g)^2}+(-B-\rho-\lambda_d-\lambda_h)(-B-\alpha_{hs}-\lambda_d-\lambda_h)(-B-\beta_{vs}-\lambda_v)\right)=0$$

or $\lambda^4+A_3\lambda^3+A_2\lambda^2+A_1\lambda+A=0$

where

$$A_3=\rho+\alpha_{hr}+\beta_{vr}+2\lambda_d+2+2\lambda_v$$

$$A_2=(\lambda_d+\lambda_h)(\rho+\lambda_d+\lambda_h)+2(\rho+2\lambda_d+2\lambda_h)\lambda_v+\lambda_v^2+\beta_{vr}(\rho+2\lambda_d+2\lambda_h+\lambda_v)+\alpha_{hr}(\rho+\beta_{vr}+\lambda_d+\lambda_h+2\lambda_v)$$

$$A_1=\beta_{vr}(\rho+\lambda_d+\lambda_h)(\alpha_{hr}+\lambda_d+\lambda_h)+(2(\lambda_d+\lambda_h)(\rho+\lambda_d+\lambda_h)+\beta_{vr}(\rho+2\lambda_d+2\lambda_h)+\alpha_{hr}(\beta_{vr}+2(\rho+\lambda_d+\lambda_h)))\lambda_v+(\rho+\alpha_{hr}+2\lambda_d+2\lambda_h)\lambda_v^2$$

$$A=\frac{1}{(N_{Tr}+g)^2}\left(-\delta^2 K_{hr\to v}K_{v\to hr}N_{Tr}N_{Vr}S_{Vr}\alpha_{hr}\beta_{vr}+(N_{Tr}+g)(\rho+\lambda_d+\lambda_h)(\alpha_{hr}+\lambda_d+\lambda_h)\lambda_v(\beta_{vr}+\lambda_v)\right)$$

$$A_3=\rho+\alpha_{hw}+\beta_{vw}+2\lambda_d+2+2\lambda_v$$

$$A_2=(\lambda_d+\lambda_h)(\rho+\lambda_d+\lambda_h)+2(\rho+2\lambda_d+2\lambda_h)\lambda_v+\lambda_v^2+\beta_{vw}(\rho+2\lambda_d+2\lambda_h+\lambda_v)+\alpha_{hw}(\rho+\beta_{vw}+\lambda_d+\lambda_h+2\lambda_v)$$

$$A_1=\beta_{vw}(\rho+\lambda_d+\lambda_h)(\alpha_{hw}+\lambda_d+\lambda_h)+(2(\lambda_d+\lambda_h)(\rho+\lambda_d+\lambda_h)+\beta_{vw}(\rho+2\lambda_d+2\lambda_h)+\alpha_{hw}(\beta_{vw}+2(\rho+\lambda_d+\lambda_h)))\lambda_v+(\rho+\alpha_{hw}+2\lambda_d+2\lambda_h)\lambda_v^2$$

$$A=\frac{1}{(N_{Tw}+g)^2}\left(-\delta^2 K_{hw\to v}K_{v\to hw}N_{Tr}N_{Vw}S_{Vw}\alpha_{hw}\beta_{vw}+(N_{Tw}+g)(\rho+\lambda_d+\lambda_h)(\alpha_{hw}+\lambda_d+\lambda_h)\lambda_v(\beta_{vw}+\lambda_v)\right)$$

$$A_3=\rho+\alpha_{hs}+\beta_{vs}+2\lambda_d+2+2\lambda_v$$

$$A_2=(\lambda_d+\lambda_h)(\rho+\lambda_d+\lambda_h)+2(\rho+2\lambda_d+2\lambda_h)\lambda_v+\lambda_v^2+\beta_{vs}(\rho+2\lambda_d+2\lambda_h+\lambda_v)+\alpha_{hs}(\rho+\beta_{vs}+\lambda_d+\lambda_h+2\lambda_v)$$

$$A_1=\beta_{vs}(\rho+\lambda_d+\lambda_h)(\alpha_{hs}+\lambda_d+\lambda_h)+(2(\lambda_d+\lambda_h)(\rho+\lambda_d+\lambda_h)+\beta_{vs}(\rho+2\lambda_d+2\lambda_h)+\alpha_{hs}(\beta_{vs}+2(\rho+\lambda_d+\lambda_h)))\lambda_v+(\rho+\alpha_{hs}+2\lambda_d+2\lambda_h)\lambda_v^2$$

$$A=\frac{1}{(N_{Ts}+g)^2}\left(-\delta^2 K_{hs\to v}K_{v\to hs}N_{Ts}N_{Vs}S_{Vs}\alpha_{hs}\beta_{vs}+(N_{Ts}+g)(\rho+\lambda_d+\lambda_h)(\alpha_{hs}+\lambda_d+\lambda_h)\lambda_v(\beta_{vs}+\lambda_v)\right)$$

Equilibrium point is local stability if eigenvalues have negative real parts. when they are according to the Routh – Hurwitz criteria:

$$A_3 > 0,$$

$$A_1 > 0,$$

$$A > 0,$$

$$A_3 A_2 A_1 - A_1^2 - A_3^2 A > 0$$

Consider condition of Routh – Hurwitz criteria as show with parameter above. Condition in above is always true . So this disease free state is local stability for $R_0 < 1$ when R_0 =

$$\left[\left(\frac{\delta^2 K_{hr\to v} K_{v\to hr} N_{tr} N_{vr} \alpha_{hr} \beta_{vr}}{(g+N_{tr})^2 (\rho+\lambda_d+\lambda_h)(\alpha_{hr}+\lambda_d+\lambda_h)\lambda_v(\beta_{vr}+\lambda_v)}\right) + \left(\frac{\delta^2 K_{hw\to v} K_{v\to hw} N_{tw} N_{vw} \alpha_{hw} \beta_{vw}}{(g+N_{tw})^2 (\rho+\lambda_d+\lambda_h)(\alpha_{hw}+\lambda_d+\lambda_h)\lambda_v(\beta_{vw}+\lambda_v)}\right) + \left(\frac{\delta^2 K_{hs\to v} K_{v\to hs} N_{ts} N_{vw} \alpha_{hs} \beta_{vs}}{(g+N_{ts})^2 (\rho+\lambda_d+\lambda_h)(\alpha_{hs}+\lambda_d+\lambda_h)\lambda_v(\beta_{vs}+\lambda_v)}\right)\right]$$

D. Endemic disease state

The endemic disease equilibrium point

$E_1 = (S^*_{hr}, E^*_{hr}, I^*_{hr}, E^*_{vr}, I^*_{vr}, S^*_{hw}, E^*_{hw}, I^*_{hw}, E^*_{vw}, I^*_{vw}, S^*_{hs}, E^*_{hs}, I^*_{hs}, E^*_{vs}, I^*_{vs})$ has characteristic equation as follows:

$$\lambda^5 + A_4\lambda^4 + A_3\lambda^3 + A_2\lambda^2 + A_1\lambda + A = 0$$

Where

$$A4 = \rho + \alpha_{hr} + 3\lambda_d + 3\lambda_h + 2\lambda_v + \beta_{vr}\left(1 + \frac{\delta^2 i_{hr} K_{hr\to v} K_{v\to hr} N_{Tr} N_{vr}}{(g+N_{tr})(\beta_{vr}+\lambda_v)(\delta i_{hr} K_{hr\to v} N_{Tr} + (g+N_{tr})\lambda_v)}\right)$$

$$A3 = -\frac{1}{(g+N_{tr})(\beta_{vr}+\lambda_v)(\delta i_{hr} K_{hr\to v} N_{Tr} + (g+N_{tr})\lambda_v)} (-(g+N_{tr})^2 \lambda_v (\beta_{vr}+\lambda_v)((\lambda_d+\lambda_h)(2\rho+3\lambda_d+3\lambda_h) + 2(\rho+3\lambda_d+3\lambda_h)\lambda_v + \lambda_v^2 + \beta_{vr}(\rho+3\lambda_d+3\lambda_h+\lambda_v) + \alpha_{hr}(\rho+\beta_{vr}+2\lambda_d+2\lambda_h+2\lambda_v)) - \delta i_{hr} K_{hr\to v} N_{Tr} (\delta K_{v\to hr} N_{vr} \beta_{vr} (\rho+\alpha_{hr}+\beta_{vr}+2\lambda_d+2\lambda_h+2\lambda_v) + (g+N_{tr})(\beta_{vr}+\lambda_v)((\lambda_d+\lambda_h)(2\rho+3\lambda_d+3\lambda_h) + 2(\rho+3\lambda_d+3\lambda_h)\lambda_v + \lambda_v^2 + \beta_{vr}(\rho+3\lambda_d+3\lambda_h+\lambda_v) + \alpha_{hr}(\rho+\beta_{vr}+2\lambda_d+2\lambda_h+2\lambda_v))))$$

$$A2 = -\frac{1}{(g+N_{tr})(\beta_{vr}+\lambda_v)(\delta i_{hr} K_{hr\to v} N_{Tr} + (g+N_{tr})\lambda_v)} (-(g+N_{tr})^2 \lambda_v (\beta_{vr}+\lambda_v)((\lambda_d+\lambda_h)^2(\rho+\lambda_d+\lambda_h) + 2(\lambda_d+\lambda_h)(2\rho+3\lambda_d+3\lambda_h)\lambda_v + (\rho+3\lambda_d+3\lambda_h)\lambda_v^2 + \beta_{vr}((\lambda_d+\lambda_h)(2\rho+3\lambda_d+3\lambda_h) + (\rho+3\lambda_d+3\lambda_h)\lambda_v) + \alpha_{hr}((\lambda_d+\lambda_h)(\rho+\lambda_d+\lambda_h) + 2(\rho+2\lambda_d+2\lambda_h)\lambda_v + \lambda_v^2 + \beta_{vr}(\rho+2\lambda_d+2\lambda_h+\lambda_v))) - \delta i_{hr} K_{hr\to v} N_{Tr} (\delta K_{v\to hr} N_{vr} \beta_{vr} ((\lambda_d+\lambda_h)(\rho+\lambda_d+\lambda_h) + 2(\rho+2\lambda_d+2\lambda_h)\lambda_v + \lambda_v^2 + \beta_{vr}(\rho+2\lambda_d+2\lambda_h+\lambda_v) + \alpha_{hr}(\rho+\beta_{vr}+\lambda_d+\lambda_h+2\lambda_v)) + (g+N_{tr})(\beta_{vr}+\lambda_v)((\lambda_d+\lambda_h)^2(\rho+\lambda_d+\lambda_h) + 2(\lambda_d+\lambda_h)(2\rho+3\lambda_d+3\lambda_h)\lambda_v + (\rho+3\lambda_d+3\lambda_h)\lambda_v^2 + \beta_{vr}((\lambda_d+\lambda_h)(2\rho+3\lambda_d+3\lambda_h) + (\rho+3\lambda_d+3\lambda_h)\lambda_v) + \alpha_{hr}((\lambda_d+\lambda_h)(\rho+\lambda_d+\lambda_h) + 2(\rho+2\lambda_d+2\lambda_h)\lambda_v + \lambda_v^2 + \beta_{vr}(\rho+2\lambda_d+2\lambda_h+\lambda_v)))))$$

$$A1 = -\frac{\delta^2 K_{hr\to v} K_{v\to hr} N_{tr} N_{vr} \alpha_{hr} \beta_{vr}}{(g+N_{tr})^2} + \rho\alpha_{hr}\beta_{vr}\lambda_d + \alpha_{hr}\beta_{vr}\lambda_d^2 + \rho\alpha_{hr}\beta_{vr}\lambda_h + 2\alpha_{hr}\beta_{vr}\lambda_d\lambda_h + \alpha_{hr}\beta_{vr}\lambda_h^2 - \frac{1}{(g+N_{tr})^2} \beta_{vr}(-(g+N_{tr})^2(\lambda_d+\lambda_h)^2(\rho+\lambda_d+\lambda_h) - \delta^2 i_{hr} K_{hr\to v} K_{v\to hr} N_{Tr} N_{vr}(\rho+\alpha_{hr}+2\lambda_d+2\lambda_h)) + (\alpha_{hr}(2(\lambda_d+\lambda_h)(\rho+\lambda_d+\lambda_h) + \beta_{vr}(\rho+2\lambda_d+2\lambda_h)) + (\lambda_d+\lambda_h)(2(\lambda_d+\lambda_h)(\rho+\lambda_d+\lambda_h) + \beta_{vr}(2\rho+3\lambda_d+3\lambda_h)))\lambda_v + (\alpha_{hr}(\rho+2\lambda_d+2\lambda_h) + (\lambda_d+\lambda_h)(2\rho+3\lambda_d+3\lambda_h))\lambda_v^2 + \frac{\delta^2 i_{hr} K_{hr\to v} K_{v\to hr} N_{Tr} N_{vr} \beta_{vr}^2 (\rho+\lambda_d+\lambda_h)(\alpha_{hr}+\lambda_d+\lambda_h)}{(g+N_{tr})(-\delta i_{hr} K_{hr\to v} N_{Tr} + (g+N_{tr})\beta_{vr})(\beta_{vr}+\lambda_v)} + (\delta^2 i_{hr} K_{hr\to v} K_{v\to hr} N_{Tr} N_{vr} \beta_{vr} ((g+N_{tr})^2 \beta_{vr}(\rho+\lambda_d+\lambda_h)(\alpha_{hr}+\lambda_d+\lambda_h) + \delta^2 i_{hr}^2 K_{hr\to v}^2 N_{Tr}^2 (\rho+\alpha_{hr}+2\lambda_d+2\lambda_h) - \delta i_{hr} K_{hr\to v}(g+N_{tr}) N_{Tr}(2(\lambda_d+\lambda_h)(\rho+\lambda_d+\lambda_h) + \beta_{vr}(\rho+2\lambda_d+2\lambda_h) + \alpha_{hr}(\beta_{vr}+2(\rho+\lambda_d+\lambda_h)))))/((g+N_{tr})^2(-\delta i_{hr} K_{hr\to v} N_{Tr} + (g+N_{tr})\beta_{vr})(\delta i_{hr} K_{hr\to v} N_{Tr} + (g+N_{tr})\lambda_v)) + (\delta^3 i_{hr} K_{hr\to v}^2 K_{v\to hr} N_{tr} N_{Tr} N_{vr} \alpha_{hr} \beta_{vr} (\delta K_{v\to hr} N_{vr} \beta_{vr} + (g+N_{tr})(\lambda_d+\lambda_h)(\beta_{vr}+\lambda_v)))/((g+N_{tr})^2((g+N_{tr})^2(\lambda_d+\lambda_h)\lambda_v(\beta_{vr}+\lambda_v) + \delta i_{hr} K_{hr\to v} N_{Tr}(\delta K_{v\to hr} N_{vr} \beta_{vr} + (g+N_{tr})(\lambda_d+\lambda_h)(\beta_{vr}+\lambda_v))))$$

$$A = (\lambda_v(\delta^2 i_{hr}^2 K_{hr\to v}^2 N_{Tr}^2 (\rho+\lambda_d+\lambda_h)(\alpha_{hr}+\lambda_d+\lambda_h)(\delta K_{v\to hr} N_{vr} \beta_{vr} + (g+N_{tr})(\lambda_d+\lambda_h)(\beta_{vr}+\lambda_v))^2 + (g+N_{tr})^2(\lambda_d+\lambda_h)^2 \lambda_v (\beta_{vr}+\lambda_v)(-\delta^2 K_{hr\to v} K_{v\to hr} N_{tr} N_{vr} \alpha_{hr} \beta_{vr} + (g+N_{tr})^2(\rho+\lambda_d+\lambda_h)(\alpha_{hr}+\lambda_d+\lambda_h)\lambda_v(\beta_{vr}+\lambda_v)) + \delta i_{hr} K_{hr\to v}(g+N_{tr}) N_{Tr}(\lambda_d+\lambda_h)(\beta_{vr}+\lambda_v)(-\delta^2 K_{hr\to v} K_{v\to hr} N_{tr} N_{vr} \alpha_{hr} \beta_{vr}(\lambda_d+\lambda_h) + 2(g+N_{tr})(\rho+\lambda_d+\lambda_h)(\alpha_{hr}+\lambda_d+\lambda_h)\lambda_v(\delta K_{v\to hr} N_{vr} \beta_{vr} + (g+N_{tr})(\lambda_d+\lambda_h)(\beta_{vr}+\lambda_v)))))/(((g+N_{tr})(\delta i_{hr} K_{hr\to v} N_{Tr} + (g+N_{tr})\lambda_v)((g+N_{tr})^2(\lambda_d+\lambda_h)\lambda_v(\beta_{vr}+\lambda_v) + \delta i_{hr} K_{hr\to v} N_{Tr}(\delta K_{v\to hr} N_{vr} \beta_{vr} + (g+N_{tr})(\lambda_d+\lambda_h)(\beta_{vr}+\lambda_v))))$$

$$A4 = \rho + \alpha_{bw} + 3\lambda_d + 3\lambda_h + 2\lambda_v + \beta_{vw}\left(1 + \frac{\delta^2 i_{bw} K_{bw\to v} K_{v\to bw} N_{Tw} N_{vw}}{(g+N_{bw})(\beta_{vw}+\lambda_v)(\delta i_{bw} K_{bw\to v} N_{Tw} + (g+N_{bw})\lambda_v)}\right)$$

$$\begin{aligned}
A3 = {} & \frac{1}{(g+N_{bw})(\beta_{vw}+\lambda_v)(\delta i_{bw} K_{bw\to v} N_{Tw} + (g+N_{bw})\lambda_v)} \\
& \Big((g+N_{bw})^2 \lambda_v (\beta_{vw}+\lambda_v)\big((\lambda_d+\lambda_h)(2\rho+3\lambda_d+3\lambda_h) + 2(\rho+3\lambda_d+3\lambda_h)\lambda_v + \lambda_v^2 + \beta_{vw}(\rho+3\lambda_d+3\lambda_h+\lambda_v) + \alpha_{bw}(\rho+\beta_{vw}+2\lambda_d+2\lambda_h+2\lambda_v)\big) + \\
& \delta i_{bw} K_{bw\to v} N_{Tw} \\
& \big(\delta K_{v\to bw} N_{vw} \beta_{vw}(\rho+\alpha_{bw}+\beta_{vw}+2\lambda_d+2\lambda_h+2\lambda_v) + (g+N_{bw})(\beta_{vw}+\lambda_v)\big((\lambda_d+\lambda_h)(2\rho+3\lambda_d+3\lambda_h) + 2(\rho+3\lambda_d+3\lambda_h)\lambda_v + \lambda_v^2 + \beta_{vw}(\rho+3\lambda_d+3\lambda_h+\lambda_v) + \alpha_{bw}(\rho+\beta_{vw}+2\lambda_d+2\lambda_h+2\lambda_v)\big)\big)\Big)
\end{aligned}$$

$$\begin{aligned}
A2 = {} & \frac{1}{(g+N_{bw})(\beta_{vw}+\lambda_v)(\delta i_{bw} K_{bw\to v} N_{Tw} + (g+N_{bw})\lambda_v)} \\
& \Big((g+N_{bw})^2 \lambda_v (\beta_{vw}+\lambda_v)\big((\lambda_d+\lambda_h)^2(\rho+\lambda_d+\lambda_h) + 2(\lambda_d+\lambda_h)(2\rho+3\lambda_d+3\lambda_h)\lambda_v + (\rho+3\lambda_d+3\lambda_h)\lambda_v^2 + \beta_{vw}((\lambda_d+\lambda_h)(2\rho+3\lambda_d+3\lambda_h) + (\rho+3\lambda_d+3\lambda_h)\lambda_v) + \\
& \quad \alpha_{bw}\big((\lambda_d+\lambda_h)(\rho+\lambda_d+\lambda_h) + 2(\rho+2\lambda_d+2\lambda_h)\lambda_v + \lambda_v^2 + \beta_{vw}(\rho+2\lambda_d+2\lambda_h+\lambda_v)\big)\big) + \\
& \delta i_{bw} K_{bw\to v} N_{Tw}\Big(\delta K_{v\to bw} N_{vw} \beta_{vw}\big((\lambda_d+\lambda_h)(\rho+\lambda_d+\lambda_h) + 2(\rho+2\lambda_d+2\lambda_h)\lambda_v + \lambda_v^2 + \beta_{vw}(\rho+2\lambda_d+2\lambda_h+\lambda_v) + \alpha_{bw}(\rho+\beta_{vw}+\lambda_d+\lambda_h+2\lambda_v)\big) + \\
& \quad (g+N_{bw})(\beta_{vw}+\lambda_v)\big((\lambda_d+\lambda_h)^2(\rho+\lambda_d+\lambda_h) + 2(\lambda_d+\lambda_h)(2\rho+3\lambda_d+3\lambda_h)\lambda_v + (\rho+3\lambda_d+3\lambda_h)\lambda_v^2 + \beta_{vw}((\lambda_d+\lambda_h)(2\rho+3\lambda_d+3\lambda_h) + (\rho+3\lambda_d+3\lambda_h)\lambda_v) + \\
& \quad\quad \alpha_{bw}\big((\lambda_d+\lambda_h)(\rho+\lambda_d+\lambda_h) + 2(\rho+2\lambda_d+2\lambda_h)\lambda_v + \lambda_v^2 + \beta_{vw}(\rho+2\lambda_d+2\lambda_h+\lambda_v)\big)\big)\Big)\Big)
\end{aligned}$$

$$\begin{aligned}
A1 = {} & -\frac{\delta^2 K_{bw\to v} K_{v\to bw} N_{bw} N_{vw} \alpha_{bw} \beta_{vw}}{(g+N_{bw})^2} + \rho\alpha_{bw}\beta_{vw}\lambda_d + \alpha_{bw}\beta_{vw}\lambda_d^2 + \rho\alpha_{bw}\beta_{vw}\lambda_h + 2\alpha_{bw}\beta_{vw}\lambda_d\lambda_h + \alpha_{bw}\beta_{vw}\lambda_h^2 - \frac{\beta_{vw}\left(-(g+N_{bw})^2(\lambda_d+\lambda_h)^2(\rho+\lambda_d+\lambda_h) - \delta^2 i_{bw} K_{bw\to v} K_{v\to bw} N_{Tw} N_{vw}(\rho+\alpha_{bw}+2\lambda_d+2\lambda_h)\right)}{(g+N_{bw})^2} + \\
& (\alpha_{bw}(2(\lambda_d+\lambda_h)(\rho+\lambda_d+\lambda_h) + \beta_{vw}(\rho+2\lambda_d+2\lambda_h)) + (\lambda_d+\lambda_h)(2(\lambda_d+\lambda_h)(\rho+\lambda_d+\lambda_h) + \beta_{vw}(2\rho+3\lambda_d+3\lambda_h)))\lambda_v + (\alpha_{bw}(\rho+2\lambda_d+2\lambda_h) + (\lambda_d+\lambda_h)(2\rho+3\lambda_d+3\lambda_h))\lambda_v^2 + \\
& \frac{\delta^2 i_{bw} K_{bw\to v} K_{v\to bw} N_{Tw} N_{vw} \beta_{vw}^2 (\rho+\lambda_d+\lambda_h)(\alpha_{bw}+\lambda_d+\lambda_h)}{(g+N_{bw})(-\delta i_{bw} K_{bw\to v} N_{Tw} + (g+N_{bw})\beta_{vw})(\beta_{vw}+\lambda_v)} + \\
& \Big(\delta^2 i_{bw} K_{bw\to v} K_{v\to bw} N_{Tw} N_{vw} \beta_{vw}\big((g+N_{bw})^2\beta_{vw}(\rho+\lambda_d+\lambda_h)(\alpha_{bw}+\lambda_d+\lambda_h) + \delta^2 i_{bw}^2 K_{bw\to v}^2 N_{Tw}^2(\rho+\alpha_{bw}+2\lambda_d+2\lambda_h) - \\
& \quad \delta i_{bw} K_{bw\to v}(g+N_{bw}) N_{Tw}(2(\lambda_d+\lambda_h)(\rho+\lambda_d+\lambda_h) + \beta_{vw}(\rho+2\lambda_d+2\lambda_h) + \alpha_{bw}(\beta_{vw}+2(\rho+\lambda_d+\lambda_h)))\big)\Big) \Big/ \left((g+N_{bw})^2(-\delta i_{bw} K_{bw\to v} N_{Tw} + (g+N_{bw})\beta_{vw})(\delta i_{bw} K_{bw\to v} N_{Tw} + (g+N_{bw})\lambda_v)\right) + \\
& \frac{\delta^3 i_{bw} K_{bw\to v}^2 K_{v\to bw} N_{bw} N_{Tw} N_{vw} \alpha_{bw} \beta_{vw}(\delta K_{v\to bw} N_{vw}\beta_{vw} + (g+N_{bw})(\lambda_d+\lambda_h)(\beta_{vw}+\lambda_v))}{(g+N_{bw})^2\left((g+N_{bw})^2(\lambda_d+\lambda_h)\lambda_v(\beta_{vw}+\lambda_v) + \delta i_{bw} K_{bw\to v} N_{Tw}(\delta K_{v\to bw} N_{vw}\beta_{vw} + (g+N_{bw})(\lambda_d+\lambda_h)(\beta_{vw}+\lambda_v))\right)}
\end{aligned}$$

The eigenvalues of endemic disease state have negative real parts ,when they are according to the Routh – Hurwitz criteria:

$$A_i > 0 \quad (i = 1, 2, 3, 4), \tag{37}$$

$$A > 0, \tag{38}$$

$$A_4A_3A_2 - A_2^2 - A_4^2A_1 > 0, \tag{39}$$

$$(A_4A_1 - A)(A_4A_3A_2 - A_2^2 - A_4^2A_1) - A(A_4A_3 - A_2)^2 - A_4^2 > 0 \tag{40}$$

Consider condition of Routh – Hurwitz criteria as show with parameter above. Condition in above is always true. We can represent conditions (39)-(40) with the figure 2 as follows:

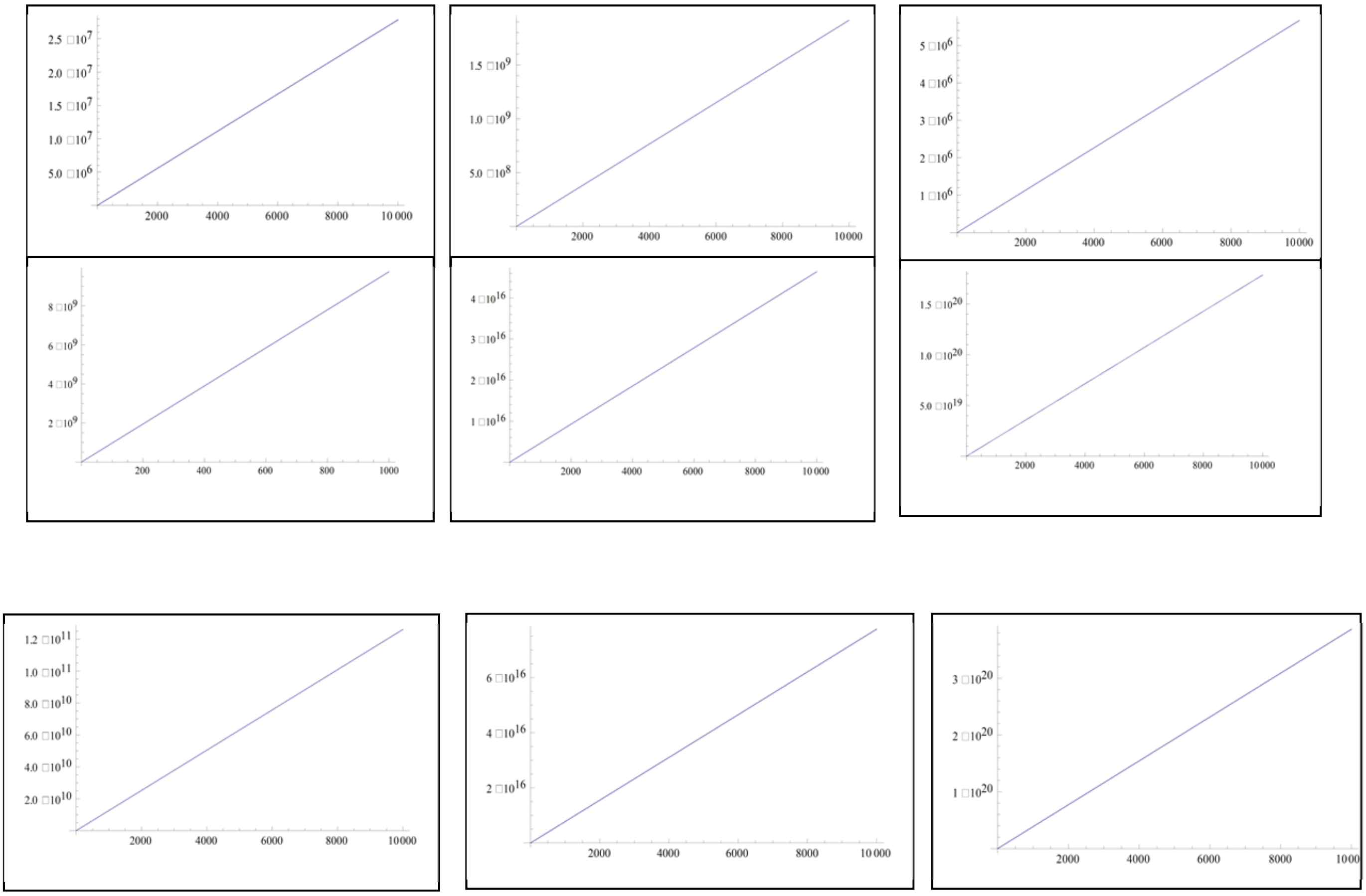

Figure 2 The condition of Routh – Hurwitz criteria(39) – (40) in endemic disease state for rainy season, winter season and summer season, respectively.

We found that all eigenvalues have negative real parts when $R_0 > 1$. This means that the endemic disease state is local stability [3] for $R_0 > 1$ when

$$R_0 = \left[\left(\frac{\delta^2 K_{hr\to v} K_{v\to hr} N_{tr} N_{vr} \alpha_{hr} \beta_{vr}}{(g + N_{tr})^2 (\rho + \lambda_d + \lambda_h)(\alpha_{hr} + \lambda_d + \lambda_h)\lambda_v (\beta_{vr} + \lambda_v)}\right) + \left(\frac{\delta^2 K_{hw\to v} K_{v\to hw} N_{tw} N_{vw} \alpha_{hw} \beta_{vw}}{(g + N_{tw})^2 (\rho + \lambda_d + \lambda_h)(\alpha_{hw} + \lambda_d + \lambda_h)\lambda_v (\beta_{vw} + \lambda_v)}\right) + \left(\frac{\delta^2 K_{hs\to v} K_{v\to hs} N_{ts} N_{vw} \alpha_{hs} \beta_{vs}}{(g + N_{ts})^2 (\rho + \lambda_d + \lambda_h)(\alpha_{hs} + \lambda_d + \lambda_h)\lambda_v (\beta_{vs} + \lambda_v)}\right)\right]$$

Results, Discussion and Conclusion

The transmission of dengue disease is analyzed by using the standard dynamical modeling method. We have shown that several standard theorems in mathematical epidemiology can be extended to this kind of SEIR model. R_0 can be expressed as

$$R_0 = \left[\left(\frac{\delta^2 K_{hr\to v} K_{v\to hr} N_{tr} N_{vr} \alpha_{hr} \beta_{vr}}{(g + N_{tr})^2 (\rho + \lambda_d + \lambda_h)(\alpha_{hr} + \lambda_d + \lambda_h)\lambda_v (\beta_{vr} + \lambda_v)}\right) + \left(\frac{\delta^2 K_{hw\to v} K_{v\to hw} N_{tw} N_{vw} \alpha_{hw} \beta_{vw}}{(g + N_{tw})^2 (\rho + \lambda_d + \lambda_h)(\alpha_{hw} + \lambda_d + \lambda_h)\lambda_v (\beta_{vw} + \lambda_v)}\right) + \left(\frac{\delta^2 K_{hs\to v} K_{v\to hs} N_{ts} N_{vw} \alpha_{hs} \beta_{vs}}{(g + N_{ts})^2 (\rho + \lambda_d + \lambda_h)(\alpha_{hs} + \lambda_d + \lambda_h)\lambda_v (\beta_{vs} + \lambda_v)}\right)\right]$$

is the basic reproductive number of the disease. If $R_0 < 1$, the disease-free equilibrium state is local stable . If $R_0 > 1$, then an endemic equilibrium is local stable. The results of this study are used for finding the condition of parameters to be the way for controlling this disease.

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